Probability Discounting and Money Pumps[*]

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ABSTRACT: In response to cases that involve tiny probabilities of huge payoffs, some argue that we ought to discount small probabilities down to zero. However, this paper shows that doing so violates Independence and Continuity, and as a result of these violations, those who discount small probabilities can be exploited by money pumps. Various possible ways of avoiding exploitation will be discussed. This paper concludes that the money pump for Independence undermines the plausibility of discounting small probabilities. Much of the discussion on Independence generalizes to other views that also violate Independence.

On standard decision theory, a rational agent always maximizes expected utility. However, this seems to lead to counterintuitive choices in cases that involve very small probabilities of huge payoffs. Consider, for example, the following case:[1]

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[1]Kosonen (2022, pp. 2-4). This case is based on Bostrom's (2009) *Pascal's Mugging*, which in turn is based on informal discussions by various people, including Eliezer Yudkowsky (2007). For criticism of Pascal's Mugging, see Hanson (2007), Baumann (2009) and Hiller and Hasan (2023). Another case that involves tiny probabilities of huge payoffs is the St. Petersburg game. See for example Peterson (2020).

Pascal's Hell: Satan offers Pascal a deal: He will create a million Graham's number of happy Earth-like planets if a coin lands on heads. But if the coin lands on tails, then everyone on Earth will suffer excruciating pain until life on Earth is no longer possible. The probability of heads is one-in-Graham's-number.

Should Pascal accept the offer? The probability of the positive payoff is tiny, so accepting the offer will almost certainly result in a negative outcome. However, as the possible payoff is enormous, Pascal is forced to conclude that the expected value of accepting the offer is positive.[2] More generally, maximizing expected utility (with unbounded utilities) leads to

Probability Fanaticism: For any probability $p > 0$ and any (finitely) good outcome o, there is some great enough outcome O such that probability p of O (and otherwise nothing) is better than certainty of o.[3]

In response to cases like this, some have argued that we ought to discount very small probabilities down to zero—let's call this *Probability Discounting*. For example, Monton (2019) argues that one ought to discount very small probabilities down to zero, while Smith (2014) argues that it is rationally permissible, but not required, to do so.[4] There are many ways of making Probability Discounting precise.[5]

[2] This may not hold if Pascal maximizes expected utility and utilities are bounded as standard axiomatizations of expected utility maximization (such as the von Neumann-Morgenstern utility theorem) require. See for example Kreps (1988, p. 64).

[3] Wilkinson (2022, p.449) and Beckstead and Thomas (2023, p. 2).

[4] Smith argues that discounting small probabilities allows one to get a reasonable expected utility for the Pasadena game. See Nover and Hájek (2004) on this game. On Smith's view, the discounting threshold could be chosen lower than any relevant probability in cases that involve finitely many possible outcomes. So, in effect, discounting small probabilities might not apply to cases involving a finite number of possible outcomes. See Hájek (2014), Isaacs (2016), Lundgren and Stefánsson (2020) and Cibinel (2023) for criticism of discounting small probabilities. Also see Beckstead (2013, Ch 6), Beckstead and Thomas (2023), Goodsell (2021), Russell and Isaacs (2021), Russell (2023) and Wilkinson (2022) for discussions of issues surrounding Probability Fanaticism.

[5] See Kosonen (2022, Ch 4) for a discussion of some possible versions of Probability Discounting.

Let $X \succsim Y$ mean that X is at least as preferred as Y. Also, let $EU(X)_{pd}$ denote the expected utility of prospect X when small probabilities have been discounted down to zero (read as 'the probability-discounted expected utility of X'). Also, let a *negligible probability* be a probability below the discounting threshold, that is, a probability that should be discounted down to zero. Then, one of the simplest versions of Probability Discounting—let's call it *Naive Discounting*—states:

> **Naive Discounting:** For all prospects X and Y, $X \succsim Y$ if and only if $EU(X)_{pd} \geq EU(Y)_{pd}$, where $EU(X)_{pd}$ and $EU(Y)_{pd}$ are obtained by conditionalizing on the supposition that some outcome of non-negligible probability occurs.

Given that Probability Discounting differs from Expected Utility Theory, it has to violate at least one of the following axioms that together entail Expected Utility Theory: Completeness, Transitivity, Independence and Continuity.[6] Violating these axioms renders probability discounters vulnerable to exploitation as there are money-pump arguments for each of these axioms.[7]

This paper shows that Probability Discounting violates Independence and Continuity. It is therefore vulnerable to exploitation in the money pumps for Independence and Continuity. Here's the structure of the paper: §1 discusses two ways in which Probability Discounting might violate Continuity. This section also shows that probability discounters are vulnerable to exploitation in a money pump for Continuity. Lastly, it discusses some ways of avoiding exploitation in that case. §2 shows that Probability Discounting violates Independence. As a result, probability discounters are vulnerable to exploitation in a money pump for Independence. §3 discusses possible ways of avoiding exploitation in the Independence Money Pump.

[6] Von Neumann and Morgenstern (1947), Jensen (1967, pp. 172–182) and Hammond (1998, pp. 152–164). This paper assumes the von Neumann-Morgenstern framework with its lotteries with given probabilities, rather than the Savage framework, where subjective probabilities must be constructed alongside utilities, requiring the use of a different and more expansive set of axioms.

[7] Gustafsson (2022). It has been argued that even agents who conform to Expected Utility Theory can be exploited in some cases with an infinite series of trade offers. Gustafsson (2022, pp. 74–77) argues that such agents can avoid exploitation if they use backward induction.

It concludes that there is no plausible way to do this. Much of the discussion in this section generalizes to other views that also violate Independence; it is not specific to Probability Discounting. The paper concludes that the Independence Money Pump greatly undermines the plausibility of Probability Discounting.

1 Continuity

This section discusses two ways in which Probability Discounting violates Continuity. First, it shows that views that discount probabilities below some discounting threshold violate Continuity. One might think it obvious that Probability Discounting violates Continuity and so is vulnerable to a simple money pump.[8] But that would be too quick: There is another way of conceiving the threshold. Instead of discounting probabilities below some threshold, probability discounters might discount probabilities up to some threshold. Nevertheless, this section shows that views that discount probabilities up to some threshold violate another version of Continuity. As a result of violating Continuity, Probability Discounting is vulnerable to exploitation in a money pump for Continuity. Some ways of avoiding exploitation in this money pump will be discussed.

1.1 The Continuity Money Pump

As mentioned earlier, Continuity is one of the axioms that together entail Expected Utility Theory. Let $X \succ Y$ mean that X is strictly preferred (or simply 'preferred') to Y. Also, let XpY be a risky prospect with a p chance of prospect X obtaining and a $1-p$ chance of prospect Y obtaining. Continuity then states the following:

> **Continuity:** If $X \succ Y \succ Z$, then there are probabilities p and $q \in (0,1)$ such that $XpZ \succ Y \succ XqZ$.

For example, suppose a coin is flipped, and an agent gets X with heads and Z with tails. Suppose further that the bias of the coin can be changed. Continuity requires

[8]See Gustafsson (2022, p. 66) for a money pump against Continuity violations.

that, with some bias, the agent prefers the coin flip to certainly getting Y, but with some other bias, the agent prefers certainly getting Y to flipping the coin.

Views that discount probabilities below some threshold violate Continuity. To see how the Continuity violation happens, consider the following prospects:[9]

Continuity Violation:

Prospect A Gives probability t of some very good outcome (and otherwise nothing).

Prospect B Certainly gives a good outcome.

Prospect C Certainly gives nothing.

Let t be the discounting threshold. Then all probabilities less than t will be discounted down to zero, but probabilities at least as great as t will not be discounted. Also, suppose that A is better than B, which is better than C; a non-negligible probability of a very good outcome (and otherwise nothing) is better than a certain good outcome, which is better than certainly getting nothing.

Next, consider the following mixed lottery (see table 1):

Prospect ApC Gives probability p of A and probability $1 - p$ of C (i.e., probability $t \cdot p$ of a very good outcome and otherwise nothing).

Given that t is the discounting threshold, t multiplied by any probability $p < 1$ must be below the discounting threshold. Consequently, $t \cdot p$ is discounted down to zero, and ApC only gives a negligible probability of a positive outcome. And, given that B certainly gives a good outcome, B must be better than ApC for all probabilities $p \in (0, 1)$. So, now we have that A is better than B, which is better than C, but B is better than ApC for all probabilities $p \in (0, 1)$—which is a violation of Continuity.

[9] Naive Discounting, Lexical Discounting, State Discounting, Stochastic Discounting and Tail Discounting (discussed in Kosonen [2022, Ch 4]) all violate Continuity in this case (if the discounting threshold is the lowest probability not discounted down to zero).

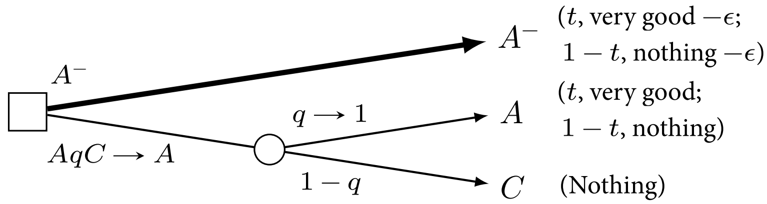
Table 1

Continuity Violation

	$p \cdot t$	$1 - p \cdot t$
ApC	Very good	Nothing
B	Good	Good

There is also a money-pump argument for Continuity. A money-pump argument intends to show that agents who violate some alleged requirement of rationality would make a combination of choices that lead to a sure loss. In so far as vulnerability to this kind of exploitation is a sign of irrationality, Probability Discounting is untenable as a theory of instrumental rationality. The money-pump argument for Continuity goes as follows:[10]

The Continuity Money Pump



$A \succ A^- \succ AqC$ for all probabilities $q \in (0, 1)$.

In this decision tree, the square represents a choice node and the circle represents a chance node. Going up at a choice node means accepting a trade and going down means refusing a trade.[11] The agent starts with AqC. AqC is arbitrarily similar to A; it results in the same outcome as A with a probability arbitrarily close to one. However, no matter how close q is to one, AqC will only give a negligible

[10] Gustafsson (2022, p. 66). Gustafsson calls this the Lexi-Pessimist Money Pump. Gustafsson (2022, p. 64) also presents another money pump against preferences that violate Continuity in the opposite way.

[11] Rabinowicz (2008, p. 152).

probability of a positive outcome. Next, the agent is offered A^- in exchange for AqC. A^- is like A except that the agent has some amount ϵ less money. A^- gives the threshold probability of a positive outcome, while AqC only gives a negligible probability of a positive outcome. Thus, the agent prefers A^- over AqC and accepts the trade. However, this means that the exploiter gets a fixed payment with only an arbitrarily small chance of having to give up anything. The situation is therefore arbitrarily close to pure exploitation.[12]

To summarize, views on which all probabilities below some discounting threshold are ignored violate Continuity, and they are therefore vulnerable to exploitation in the Continuity Money Pump.

1.2 Mixture Continuity

The previous Continuity violation happens because the discounting threshold multiplied by any probability below one results in a probability below the discounting threshold. This happens because the discounting threshold is the lowest probability not discounted down to zero. Hence, the set of non-discounted values is closed (i.e., it is an interval of the form [t, 1]).

One may think that Probability Discounting obviously violates Continuity, but that is because one is thinking of the threshold as the lowest probability not discounted down to zero. However, instead of the threshold being the lowest probability not discounted, it might be the highest probability that is discounted. If so, it is not so obvious that Probability Discounting violates Continuity.

In that case, there is no lowest non-negligible probability, and the set of non-discounted values is open on one side (i.e., it is an interval of the form (t, 1]). Consequently, A will only have positive probability-discounted expected utility if it gives

[12]Some might object that the requirement not to come arbitrarily close to being money-pumped is begging the question against Continuity-violators in a way that the requirement not to be money-pumped need not be. After all, Continuity is meant to rule out infinite and infinitesimal value differences. See Gustafsson (2022, p. 65) on this point. However, probability discounters need not believe in infinite goods or bads, so the use of the Continuity Money Pump is on a more secure ground in this case.

at least a $t+\varepsilon$ probability of a positive outcome, where ε is positive but arbitrarily close to zero. But in that case, one can always find some probability p (that may be very close to one), such that $p(t+\varepsilon) > t$. In other words, for all probabilities above the discounting threshold, there is some probability p such that their product is still above the discounting threshold. Consequently, Probability Discounting can avoid the previous violation of Continuity by letting the discounting threshold be the highest probability discounted down to zero.

However, this view violates another version of Continuity:

> **Mixture Continuity:** For all prospects X, Y and Z, the set of probabilities $\{ p \in [0,1] \}$ with property $XpZ \succsim Y$ and the set of probabilities $\{ q \in [0,1] \}$ with property $Y \succsim XqZ$ are closed.[13]

In effect, this principle states that if prospect XpZ is at least as good as prospect Y with some probability p, then there must be some highest and some lowest probability with which XpZ is at least as good as Y. (Similarly, if prospect Y is at least as good as prospect XqZ, then there must be some highest and some lowest probability with which Y is at least as good as XqZ).

To see how the view under consideration violates Mixture Continuity, consider the following prospects:[14]

> **Mixture Continuity Violation:**
>
> *Prospect A* Certainly gives a very good outcome.

[13]This is axiom 2 in Herstein and Milnor (1953, p. 293). Another way to state Mixture Continuity is as follows: If $\lim_{i\to\infty} p_i = p$ and each $Xp_iZ \succsim Y$, then $XpZ \succsim Y$. Similarly, if $\lim_{i\to\infty} p_i = p$ and $Y \succsim Xp_iZ$, then $Y \succsim XpZ$.

[14]This case is also a violation of the following version of Continuity that can be derived from Mixture Continuity (Herstein and Milnor, 1953, pp. 293–294):

> **Continuity (weak-preference):** If $X \succsim Y \succsim Z$, then there is a probability $p \in (0,1)$ such that $Y \sim XpZ$.

In Mixture Continuity Violation, A is better than B, which is better than C. However, there is no probability $p \in (0,1)$ such that $B \sim ApC$. When $p > t$, ApC is better than B (we can suppose so); when $p \leq t$, ApC is worse than B because it only gives a negligible probability of a positive outcome.

Prospect B Certainly gives a good outcome.

Prospect C Certainly gives nothing.

Again, A is better than B, which is better than C. Moreover, suppose that the very good outcome is sufficiently great so that ApC is at least as good as B for all $p > t$. Given that t is discounted down to zero, it is not the case that AtC is at least as good as B. So, there is no lowest probability p with which ApC is at least as good as B. For all $p > t$, ApC is at least as great as B; when $p = t$, ApC is worse than B. This is a violation of Mixture Continuity.[15]

Furthermore, even though this view avoids the first Continuity violation, it is still vulnerable to the Continuity Money Pump. Let $A_{t+\varepsilon}$ be a prospect that gives probability $t+\varepsilon$ of a very good outcome (and otherwise it gives nothing). $A_{t+\varepsilon}$ has positive probability-discounted expected utility for all $\varepsilon > 0$, no matter how close ε is to zero. Also, let $A_{t+\varepsilon}pC$ be a prospect that gives probability $p(t+\varepsilon)$ of a very good outcome (and otherwise it gives nothing). If ε is very close to zero, $A_{t+\varepsilon}pC$ will only have positive probability-discounted expected utility if p is very close to one—otherwise the probability of a positive outcome would be at most t, and thus, discounted down to zero. As ε can be arbitrarily close to zero, $A_{t+\varepsilon}pC$ does not have positive probability-discounted expected utility with probabilities arbitrarily close to one; as long as $p(t+\varepsilon)$ is at most t, $A_{t+\varepsilon}pC$ is at most marginally better than nothing. Consequently, even when p is very close to one, probability discounters would be willing to pay some fixed amount in order to trade $A_{t+\varepsilon}pC$ for $A_{t+\varepsilon}$ in the Continuity Money Pump.

So, if we fix p, no matter how close to one, we can find a version of the Continuity Money Pump where the exploiter wins with probability p as long as we choose ε sufficiently close to zero. Therefore, an exploiter can get a fixed payment (up to the value of $A_{t+\varepsilon}$) from the agent with only an arbitrarily small chance ($1-p$) of having to give up anything.

[15] As before, Naive Discounting, Lexical Discounting, State Discounting, Stochastic Discounting and Tail Discounting all violate Mixture Continuity in this way (if the discounting threshold is the highest probability discounted down to zero).

To summarize, views on which probabilities up to some discounting threshold are ignored violate Mixture Continuity. They are also vulnerable to exploitation in the Continuity Money Pump.

1.3 Vulnerability to the Continuity Money Pump

Probability discounters are vulnerable to exploitation in the Continuity Money Pump because arbitrarily small increases in probability, from just below the discounting threshold to just above it, can make a large difference to the value of a prospect. One partial solution would be to reduce probabilities just above the discounting threshold, but not all the way down to zero.[16] Probability discounters would still choose A^- in the Continuity Money Pump. But they would not be willing to pay as much for it as they would without reducing probabilities above the discounting threshold.

However, even if probabilities above the discounting threshold are reduced, it may be possible to compensate for those reduced probabilities by increasing the payoff at stake. So, probability discounters would still pay a significant sum to get A^- instead of AqC. Nevertheless, unlike in the Independence Money Pump (discussed later), at least probability discounters would be paying for something, namely, for a small increase in the probability of a positive outcome (from just below the discounting threshold to just above it). Therefore, this money pump is not as worrisome as the Independence Money Pump.[17]

Furthermore, it might be argued that agents who maximize expected utility are also vulnerable to schemes that are arbitrarily close to exploitation.[18] They will ac-

[16]Reducing probabilities just above the discounting threshold is discussed in Monton (2019, §6.3).

[17]Resolute Choice and Self-Regulation (discussed later) do not help in the Continuity Money Pump because this money pump is not dynamic like the Independence Money Pump; it only involves one choice node. Also, Avoid Exploitable Plans and Avoid Dominated Plans (discussed later) do not help avoid exploitation because A^- is not dominated by AqC as these prospects give slightly different probabilities.

[18]See for example Bostrom's (2009) Pascal's Mugging.

cept gambles that are arbitrarily close to a certain loss as long as the payoff in the small-probability state is great enough. However, unlike probability discounters, they will not pay a fixed amount for arbitrarily small changes in probabilities. The Continuity Money Pump illustrates how probability discounters, who wish to ignore tiny probabilities, do care a great deal about tiny *changes* in probabilities.[19,20]

To summarize, this section discussed two ways in which Probability Discounting might violate Continuity. First, it showed that views that discount probabilities below some threshold violate Continuity. Next, it showed that views that discount probabilities up to some threshold violate Mixture Continuity. Preferences that violate Continuity in these ways are vulnerable to exploitation by a money pump. However, the Continuity Money Pump is not as worrisome as the money pump for Independence because, in the former, the agent is at least paying for something: a small increase in probability from just below the discounting threshold to just above it. Next, I will discuss the Independence Money Pump, which is a case of pure exploitation.

2 Independence

This section shows that Probability Discounting violates Independence. Then, it shows how violating Independence renders probability discounters vulnerable to exploitation in a money pump for Independence. §3 discusses possible ways of avoiding exploitation in this case.

[19] Similarly, Beckstead and Thomas (2023, §3.3) point out that Probability Discounting implies the following principle:

> **Threshold Timidity:** There is some discounting threshold such that, for any finite, positive payoffs x and y, getting x with probability below the threshold is never better than getting y with probability above the threshold—no matter how much better x is than y and no matter how close together the two probabilities may be.

Threshold Timidity states that, close to the threshold, decreasing probability is infinitely more important than increasing expected utility.

[20] One possible response to the objection that probability discounters care about arbitrarily small changes in probabilities is that the discounting threshold is vague.

2.1 A violation of Independence

To see how Probability Discounting violates Independence, consider the following prospects:[21]

> *Prospect A* Gives probability q of some very good outcome (and otherwise nothing).
>
> *Prospect B* Certainly gives a good outcome.
>
> *Prospect C* Certainly gives nothing.

Let q be a probability that is above the discounting threshold but less than one. Suppose that the very good outcome is sufficiently great so that A is better than B. Next, consider the following mixed lotteries (see table 2):

> **Independence Violation:**
>
> *Prospect ApC* Gives probability p of A and probability $1-p$ of C (i.e., probability $p \cdot q$ of a very good outcome and otherwise nothing).
>
> *Prospect BpC* Gives probability p of B and probability $1-p$ of C (i.e., probability p of a good outcome and otherwise nothing).

Given that B certainly gives a positive outcome, while A gives only a probability q of a positive outcome, we can mix A and B with C so that A mixed with C (i.e., ApC) gives only a negligible probability of a positive outcome but B mixed with C (i.e., BpC) gives a non-negligible probability of a positive outcome. This is so because there must be some probability $p \in (0,1)$ such that the result of q multiplied by p is below the discounting threshold, but p itself is above that threshold. Suppose that the outcomes in question are monetary and that the utility of money equals the monetary amount. Then, there must be some p such

[21] Naive Discounting, Lexical Discounting, State Discounting, Stochastic Discounting and Tail Discounting (discussed in Kosonen [2022, Ch 4]) all violate Independence in this case.

that the probability-discounted expected utility of ApC is zero, but BpC has positive probability-discounted expected utility. In that case, Probability Discounting judges ApC to be worse than BpC.

TABLE 2
A VIOLATION OF INDEPENDENCE

	p		$1-p$
	$p \cdot q$	$p(1-q)$	$1-p$
ApC	Very good	Nothing	Nothing
BpC	Good	Good	Nothing

Now, we have that A is better than B, but ApC is worse than BpC for some $p \in (0, 1]$. This is a violation of the following axiom of Expected Utility Theory:

Independence: If $X \succ Y$, then $XpZ \succ YpZ$ for all probabilities $p \in (0, 1]$.[22]

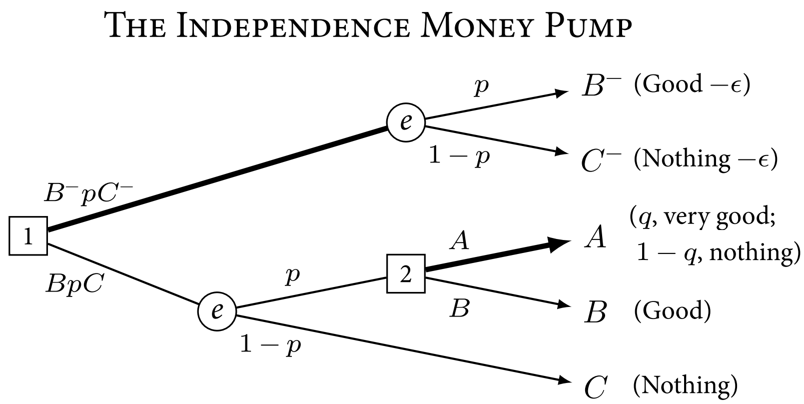
Informally, Independence is the idea that a lottery's contribution to the value of a mixed lottery does not depend on the other lotteries. The previous violation of Independence happens because, by mixing gambles together, one can reduce the probabilities of states or outcomes until their probabilities end up below the discounting threshold. As A gives a lower probability of a positive outcome than B does, with some values of p, ApC only gives a negligible probability of a positive outcome, while BpC still gives a non-negligible probability.

2.2 The Independence Money Pump

Violating Independence renders probability discounters vulnerable to exploitation in the Independence Money Pump. The case is as follows:[23]

[22] Jensen (1967, p. 173).

[23] This money pump is from Gustafsson (2021, p. 31n21; 2022, p. 57). Also see Hammond (988a, pp. 292–293; 988b, pp. 43–45).



$A \succ B$, and $BpC \succ B^-pC^- \succ ApC$.

The agent starts with prospect BpC: probability p of a good outcome and otherwise nothing. At node 1, the agent is offered a trade from BpC to B^-pC^-. B^-pC^- is just like BpC except that the agent has less money. If the agent turns down this trade and BpC results in the agent going up at chance node e, then at node 2, the agent will be offered a trade from B (certain good outcome) to A (probability q of a very good outcome and otherwise nothing). Both chance nodes depend on the same chance event e.

The agent can use *backward induction* to reason about this decision problem. This means that the agent considers what they would choose at later choice nodes and then takes those predictions into account when making choices at earlier choice nodes.[24] As the agent prefers A to B, they would accept the trade at node 2. By using backward induction at node 1, the agent can reason that the prospect of turning down the trade at node 1 is effectively ApC, and the prospect of accepting the trade is B^-pC^-. Given that the agent prefers BpC to ApC, it seems plausible that there is some price ϵ that they would be willing to pay to get the former instead of the latter. So, the agent pays that price and ends up with B^-pC^-. But they have ended up with B^-pC^- even though they could have kept BpC for free had they gone down

[24] Selten (1975) and Rosenthal (1981, p. 95).

at both choice nodes. Therefore, they have given up money for the exploiter.[25]

To summarize, this section showed that Probability Discounting violates Independence. This Independence violation happens because, by mixing gambles together, one can reduce the probabilities of states or outcomes until their associated probabilities are below the discounting threshold. As a result of violating Independence, probability discounters are vulnerable to exploitation in the Independence Money Pump. The next section discusses some possible ways of avoiding exploitation in this decision problem.

3 Avoiding exploitation in the Independence Money Pump

This section discusses how probability discounters (and others who violate Independence) can avoid exploitation in the Independence Money Pump. It argues that none of the standard views, such as Resolute Choice and Self-Regulation, work. It also argues that even if vulnerability to exploitation is not a sign of irrationality, Probability Discounting has untenable implications in a version of the Independence Money Pump that might result in a loss.

3.1 Self-Regulation

One decision policy that has been proposed as a solution to money pumps is *Self-Regulation*.[26] Self-Regulation forbids (if possible) choosing options that may lead via a rationally permissible route to a final outcome that is unchoiceworthy by the agent's own lights.[27] The idea is that one ought not choose options that may (fol-

[25] Also, as the chance nodes depend on the same event e, going up at node 1 is statewise dominated by going down at both choice nodes. See Gustafsson (2022, pp. 57–58).

[26] Self-Regulation helps avoid exploitation in money pumps against cyclic preferences. See Ahmed (2017). See Gustafsson (2022, pp. 15–19) for criticism of Self-Regulation: It conflicts with a very minimal form of backward induction and Stochastic Dominance.

[27] Ahmed (2017, p. 1001).

lowing one's preferences) lead to an outcome that one would not choose in a direct choice of all final outcomes. Unlike Resolute Choice (discussed later), Self-Regulation is forward-looking. When an agent's present choices determine the options available to them in the future, they should now choose so that their future choices lead to what they now consider acceptable in light of what is now available.[28] If the agent now wants to avoid some final outcome O, and they know what they are going to do at later choice nodes, then they should (if possible) now choose in such a way that, given those later choices, they will not end up with O.[29]

Self-Regulation in its original formulation does not help in the Independence Money Pump, as it was intended for money pumps that do not involve chance.[30] The Independence Money Pump involves chance nodes, so the agent does not know what the final holding will be. One way to adapt Self-Regulation to cases that involve chance is to apply it to plans. A plan specifies a sequence of choices to be taken by an agent at each choice node that can be reached from that node while following this specification. Self-Regulation with respect to plans then states the following:

> **Self-Regulation for Plans (i.e., Avoid Unchoiceworthy Plans):** If possible, one ought not choose options that may (following one's preferences) lead one to follow a plan that one would not choose in a direct choice of all plans (assuming one was able to commit to following some available plan).

Self-Regulation for Plans is a partial characterization of what it means to follow one's preferences: It involves, if possible, not choosing options that may, following one's preferences, lead one to follow an unchoiceworthy plan. A forward-looking choice rule C is self-regulating if and only if it tells you, at each node x, to choose a

[28] Ahmed (2017, p. 1013).

[29] Ahmed (2017, p. 1003).

[30] Rabinowicz (2021, n. 13) writes: "[H]e [Ahmed, 2017] only shows how self-regulation allows the agents with cyclic preferences to avoid dynamic inconsistency. It is unclear whether and how this approach can be extended to agents who violate Independence."

safe option whenever one is available. An option is 'safe' if and only if subsequently acting in accordance with C will lead you to follow a plan that is permissible at x.

The available plans at node 1 of the Independence Money Pump correspond to prospects ApC, BpC and B^-pC^-. One would not choose ApC or B^-pC^- in a direct choice between these plans. Therefore, one should not (if possible) choose any option that may lead via a rationally permissible route to one following ApC or B^-pC^-. However, both accepting and rejecting the trade at node 1 of the Independence Money Pump lead the agent to follow one of these plans via rationally permissible routes. Rejecting the offer leads one to follow ApC; accepting the offer leads one to follow B^-pC^-. So, Self-Regulation for Plans is silent in this case because it is not possible to make choices that do not lead to unchoiceworthy plans via rationally permissible routes. Thus, Self-Regulation for Plans does not help avoid exploitation in the Independence Money Pump. To get out of trouble, probability discounters need to find some other decision policy.[31]

3.2 Avoid Exploitable Plans

Instead of accepting Self-Regulation for Plans, one might restrict the set of forbidden plans and accept the following decision rule:

> **Avoid Exploitable Plans:** If possible, one ought not choose options that may (following one's preferences) lead one to pay for a plan that one could keep for free.

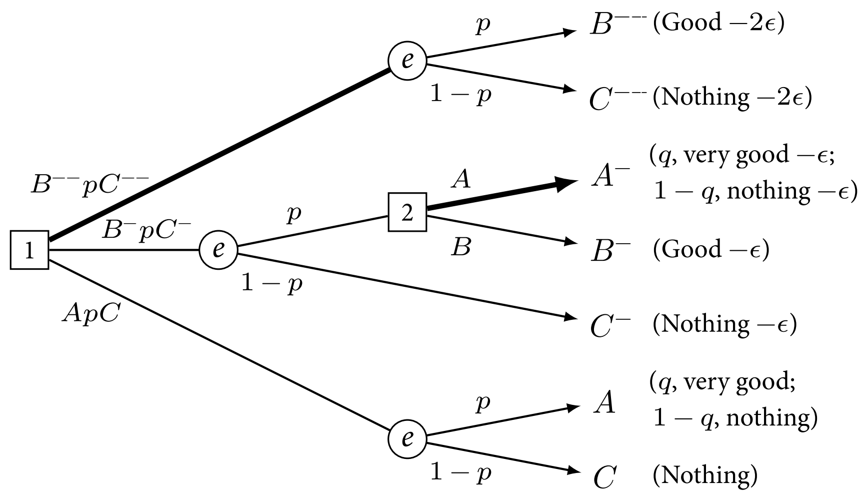
Avoid Exploitable Plans forbids accepting the trade at node 1 of the Independence Money Pump because accepting it would be paying for something that one could keep for free. However, Avoid Exploitable Plans does not forbid choosing A over B at node 2 because doing so would not be paying for a plan that one could keep for free. Thus, at node 2, an agent using Avoid Exploitable Plans would choose A

[31]Note that Self-Regulation for Plans does not help others who violate Independence avoid exploitation in the Independence Money Pump either; the discussion is not specific to Probability Discounting.

over B, given that they prefer the former. So, if one uses Avoid Exploitable Plans, one can avoid getting money pumped in the Independence Money Pump.

However, in the following decision problem, someone using Avoid Exploitable Plans would pay a higher price for something they could have obtained cheaper:[32]

The Three-Way Independence Money Pump



$A \succ A^{-} \succ B^{-} \succ B^{--}$, and $B^{-}pC^{-} \succ B^{--}pC^{--} \succ ApC \succ A^{-}pC^{-}$.

In this case, the agent starts with ApC. At node 1, they are offered $B^{-}pC^{-}$ and $B^{--}pC^{--}$. $B^{--}pC^{--}$ is like $B^{-}pC^{-}$ except that the agent has even less money (-2ϵ vs. $-\epsilon$). If the agent chooses $B^{-}pC^{-}$ and ends up in node 2, then they are offered A^{-} in exchange for B^{-}. As the agent prefers A^{-} to B^{-}, they would accept the offer. So, choosing $B^{-}pC^{-}$ at node 1 means effectively choosing $A^{-}pC^{-}$, given one's later choices. An agent who uses Avoid Exploitable Plans would therefore

[32]It might be objected that expected utility maximizers must also end up worse off than they could have been in some cases with an infinite series of trades. See for example Gustafsson (2022, pp. 74–77). However, expected utility maximizers might argue that there is a difference between not choosing the best option and paying more than one needs to, as the latter involves freely giving up what one already possesses while the former does not. But this kind of status quo bias may not be rationally justified.

choose $B^{--}pC^{--}$; they prefer $B^{--}pC^{--}$ over ApC and $A^{-}pC^{-}$, and choosing it does not mean the agent is paying for something they could keep for free (as the agent starts with ApC). However, as $B^{-}pC^{-}$ is also available, the agent has paid more than they needed to for BpC. They could have paid just ϵ instead of 2ϵ had they chosen $B^{-}pC^{-}$ at node 1 (and then kept B^{-} at node 2).

3.3 Avoid Dominated Plans

The focus on avoiding monetary exploitation may be misplaced. Instead, one might prefer adopting a decision rule that forbids all dominated plans whether or not they involve monetary exploitation:[33]

> **Avoid Dominated Plans:** If possible, one ought not choose options that may (following one's preferences) lead one to pay more for a plan that one could obtain for less money.

Avoid Dominated Plans forbids accepting the offer at node 1 of the Independence Money Pump because $B^{-}pC^{-}$ is dominated by BpC. Also, with this decision rule, one can refuse both offers of the Three-Way Independence Money Pump and keep ApC. One should refuse $B^{--}pC^{--}$ because it is dominated by $B^{-}pC^{-}$. And, one should refuse $B^{-}pC^{-}$ because choosing it means one is effectively choosing $A^{-}pC^{-}$, and $A^{-}pC^{-}$ is dominated by ApC. So, one should keep ApC. Avoid Dominated Plans thus allows an agent to avoid paying too much in this decision problem.

However, Avoid Dominated Plans seems a too narrow decision policy. Self-Regulation for Plans forbids choices that lead to plans that are unchoiceworthy by the agent's own lights. In contrast, Avoid Dominated Plans only forbids choices that lead to dominated plans but allows choices that lead to unchoiceworthy plans (such

[33] Avoid Dominated Plans is formulated in terms of monetary dominance: One should avoid plans that one can obtain for less money. But one should surely avoid plans that are dominated in other ways as well. More generally, one should avoid plans that are dominated with respect to anything valuable.

as ApC). It seems difficult to motivate such a decision policy. Why would it be irrational to choose an option that leads to a dominated plan (such as $B^{--}pC^{--}$) but not irrational to choose an option that leads to an unchoiceworthy plan (such as ApC)? Allowing the latter but forbidding the former seems arbitrary. Moreover, it leads one to something that is worse than the dominated plan, namely, ApC.

Furthermore, if we change the probabilities in the Independence Money Pump slightly, then Avoid Dominated Plans no longer avoids exploitation, at least entirely. Now, instead of $B^{-}pC^{-}$, the agent faces $B^{-}qC^{-}$, where q is arbitrarily close to p (and $q < p$). Then, given that $B^{-}qC^{-}$ and BpC do not give the exact same probabilities of the relevant outcomes, Avoid Dominated Plans no longer forbids accepting the trade at node 1; it is not the case that $B^{-}qC^{-}$ is like BpC except that the agent has less money, so Avoid Dominated Plans is silent. Consequently, a probability discounter who uses Avoid Dominated Plans will choose $B^{-}qC^{-}$ even though they could have kept BpC for free, and q is arbitrarily close to p. They have therefore given a fixed payment ϵ for an arbitrarily small increase in the probability of a positive outcome. So, Avoid Dominated Plans is vulnerable to a scheme that is arbitrarily close to exploitation.[34,35]

3.4 Resolute Choice

Self-Regulation (and related principles) do not help probability discounters avoid monetary exploitation. But perhaps Resolute Choice will? A resolute agent chooses in accordance with any plan they have adopted earlier as long as nothing unexpected has happened since the adoption of the plan.[36] If one accepts Resolute Choice, one can make a plan that one will not trade B for A in node 2 of the Independence Money Pump. Even though one would usually prefer A over B, one is

[34] This objection also applies to Avoid Exploitable Plans.

[35] Similarly as Self-Regulation for Plans, Avoid Dominated Plans and Avoid Exploitable Plans do not help others who violate Independence avoid exploitation in the Independence Money Pump either; as before, the discussion is not specific to Probability Discounting.

[36] Strotz (1956) and McClennen (1990, pp. 12–13). See Steele (2007), Steele (2018) and Gustafsson (2022, pp. 66–74) for criticism of Resolute Choice.

now committed to keeping B regardless. Consequently, one can safely refuse the trade at node 1, as one is then choosing BpC over $B^{-}pC^{-}$; one will not get money pumped nor choose the inferior prospect ApC.

However, combining Probability Discounting with Resolute Choice gives untenable results in another case. Consider the following prospects:

> *Prospect A* Certainly gives nothing.
>
> *Prospect B* Gives probability r of some very bad outcome and probability $1 - r$ of a barely positive outcome.
>
> *Prospect C* Certainly gives a barely positive outcome.

Let r be a probability above the discounting threshold but less than $1 - r$ (i.e., less than 0.5). Suppose the very bad outcome in B is sufficiently bad so that A is better than B; certainly getting nothing is better than a non-negligible chance of a very bad outcome and otherwise a barely positive outcome.

Next, consider the following mixed lotteries (see table 3):

> **Independence Violation (Negative):**
>
> *Prospect ApC* Gives probability p of A and probability $1-p$ of C (i.e., probability p of nothing and otherwise a barely positive outcome).
>
> *Prospect BpC* Gives probability p of B and probability $1 - p$ of C (i.e., probability $p \cdot r$ of a very bad outcome and otherwise a barely positive outcome).

Given that r is less than $1 - r$, there must be some (relatively small) probability $p \in (0, 1)$ such that the result of r multiplied by p is below the discounting threshold, but the result of $1-r$ multiplied by p is above the discounting threshold. In that case, the possibility of obtaining a very bad outcome with BpC is ignored. However, given that $p(1 - r)$ is above the discounting threshold, BpC gives a greater probability of a barely positive outcome than ApC.[37] Consequently, BpC is better than ApC.

But now we have a similar violation of Independence as before: A is better than B, but BpC is better than ApC.[38] This violation of Independence happens because the probability of a very bad outcome is above the discounting threshold in B but below the discounting threshold in the mixed lottery BpC. Thus, the possibility of a very bad outcome is not ignored in B, but it is ignored in BpC.

Table 3
Independence Violation (Negative)

	p		$1-p$
	$p \cdot r$	$p(1-r)$	$1-p$
ApC	Nothing	Nothing	Barely positive
BpC	Very bad	Barely positive	Barely positive

Let's go back to Resolute Choice and the Independence Money Pump. Recall that a probability discounter who uses Resolute Choice would commit to keeping B in node 2 of the Independence Money Pump (and thus avoids getting money pumped). In other words, they would commit to keeping a prospect that certainly gives a good outcome instead of trading it for a non-negligible chance of a very good outcome (and otherwise nothing). This does not seem untenable; one might bite the bullet and accept this implication. However, the same is not true in the case discussed now.

[37] This is true whether one ignores very-small-probability outcomes or states. Thus, this argument applies to all Naive Discounting, Lexical Discounting, State Discounting, Stochastic Discounting and Tail Discounting (discussed in Kosonen 2022, §4). If one ignores very-small-probability states and we take columns 2–4 in table 3 to correspond to states, then one ought to ignore column 2 (and not ignore columns 3 and 4). If one ignores very-small-probability outcomes, one ought to ignore the possibility of obtaining a very bad outcome with BpC (and not ignore the possibilities of the other outcomes). Either way, BpC gives a greater probability of a barely positive outcome than ApC.

[38] This violation of Independence is similar to the one discussed in Kosonen (2022, Ch 4).

This time the resolute choice seems unreasonable: The agent would choose a prospect that gives a non-negligible probability r of some very bad outcome and otherwise a barely positive outcome over the certainty of getting nothing. Earlier, we assumed that r is above the discounting threshold but less than $1 - r$. So, it could be, for example, 0.49. Then, the agent would choose a prospect that gives a 0.49 probability of a very bad outcome and otherwise a barely positive outcome over certainly getting nothing. Furthermore, note that the very bad outcome can be arbitrarily bad, while the barely positive outcome can be arbitrarily close to getting nothing. No reasonable theory recommends making this choice.

Appeals to Resolute Choice seem to provide a general means of answering dynamic choice arguments against various patterns of preferences. However, Probability Discounting combined with Resolute Choice leads to disastrous results. Thus, Probability Discounting combined with Resolute Choice is untenable as a theory of instrumental rationality. So, although Resolute Choice may help others who violate Independence avoid exploitation in the Independence Money Pump, it does not help probability discounters.

3.5 How worrisome are the Independence Money Pumps?

Probability discounters might argue that these money pumps are not worrisome because, for example, the agent only really faces prospects ApC and $B^{-}pC^{-}$ at node 1 of the Independence Money Pump, given that they would choose A at node 2.[39] Thus, given the agent's preferences, in a way BpC is not even available to the agent. So, by choosing $B^{-}pC^{-}$, the agent does not end up paying for something they could have kept for free. However, a money-pump argument is supposed to show that a given set of preferences is irrational because they lead to the agent paying for something they could have kept for free (if they had some other preferences). Therefore, it is not an adequate defense of those preferences that, given those preferences, the agent did not have any other option but to pay for something

[39] See Levi (1997, p. 82n10) and Levi (2002, p. S241) for this point.

they could have kept for free. The target of the money pump is the structure of preferences.[40] If one's preferences lead one to pay for something one could have kept for free (if one had some other preferences), then the money pump has succeeded in showing that those preferences are irrational.

Furthermore, even if being exploited is not a sign of irrationality as this argument claims, the violation of Independence in the case that includes negative payoffs (see table 3) is worrisome independently of the exploitation it leads to. The reason for this is that the agent would choose to lock in a choice of keeping B (at node 2) if that was somehow possible at node 1.[41] This means they would lock in a choice of a prospect that gives a 0.49 probability of a very bad outcome and otherwise a barely positive outcome over certainly getting nothing. This seems irrational. So, even if probability discounters do not accept Resolute Choice, they would still make the same choice of B over A if offered the chance to lock in the choice at node 1.[42] This makes Probability Discounting less plausible as a theory of instrumental rationality.[43]

To conclude, this section discussed possible ways of avoiding exploitation in the Independence Money Pump.[44] First, it showed that Self-Regulation for Plans does not avoid exploitation in the Independence Money Pump. An agent who uses Avoid Exploitable Plans would pay too much for a plan in the Three-Way Independence Money Pump. Avoid Dominated Plans solves the Three-Way Independence

[40] Steele (2010, p. 474) and Gustafsson (2022, p. 8n. 29, 14).

[41] The agent would, therefore, also avoid costless information. More generally, agents who violate Independence avoid costless information. See for example Wakker (1988), Hilton (1990) and Machina (1989, p. 1638–1639).

[42] Kosonen (2022, Ch 4).

[43] It is worth pointing out that, independently of Probability Discounting, agents with unbounded utilities are also vulnerable to money pumps because they violate countable generalizations of the Independence axiom. See Russell and Isaacs (2021).

[44] Another decision policy that might help probability discounters is *Myopic Choice*. Myopic Choice advises an agent to choose at each choice node the option that currently seems best with no regard to what one will choose at later choice nodes. See Strotz (1956) and von Auer (1998, p. 111). However, probability discounters (and others who violate Independence) who use Myopic Choice are vulnerable to exploitation in another money pump. See Hammond (988a, p. 293).

Money Pump, but it is vulnerable to a scheme that is arbitrarily close to pure exploitation. Finally, Resolute Choice leads to untenable results in the negative version of the Independence Money Pump.

It was also argued that locking in the choice of B over A at node 2 of the negative version of the Independence Money Pump is irrational—and that this is something probability discounters would do regardless of whether they accept Resolute Choice or not. So, even if vulnerability to exploitation is not a sign of irrationality, Probability Discounting has untenable implications in the negative version of the Independence Money Pump. All in all, what we learn from these money pumps is that the various possible ways of avoiding exploitation do not ultimately work.[45] In addition, we learn that Probability Discounting gives untenable implications even if exploitation is not a sign of irrationality.

4 Conclusion

Probability Discounting is one way to avoid fanatical choices in cases that involve tiny probabilities of huge payoffs. However, it faces some serious problems.

First, this paper discussed two ways in which Probability Discounting might violate Continuity. It was shown that views that discount probabilities below some discounting threshold violate Continuity. Also, it was shown that views that discount probabilities up to some discounting threshold violate Mixture Continuity. As a result of these Continuity violations, Probability Discounting is vulnerable to exploitation in the Continuity Money Pump.

In addition to violating Continuity, Probability Discounting also violates Independence. This renders probability discounters vulnerable to exploitation in the

[45]The money pump arguments against Probability Discounting should be persuasive even for those who reject Independence for other reasons (e.g., due to the Allais paradox), as they might use Resolute Choice to avoid exploitation in the money pumps for Independence. However, as argued above, this solution is not available to probability discounters. In contrast, the Continuity Money Pump is not particularly worrying for probability discounters who already violate Continuity for other reasons.

Independence Money Pump. Some possible ways of avoiding exploitation were discussed. However, these either failed to avoid exploitation in some version of the Independence Money Pump or they had otherwise untenable implications. It was also argued that even if vulnerability to exploitation is not a sign of irrationality, Probability Discounting has untenable implications in the negative version of the Independence Money Pump.

To conclude, this paper has shown that Probability Discounting is vulnerable to exploitation in the money pumps for Independence and Continuity. The former is more worrisome than the latter, and it is difficult to see how Probability Discounting can respond to this challenge.

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